Approximating Traveling Salesman Problems using Algebraic Techniques

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2:30-6 pm, Saturday October 8, 2016

We will give an overview of the recent progress in our understanding of traveling salesman problems as well as an introduction to the underlying algebraic techniques. We will cover the following:

- Thin trees, maximum entropy convex programs, spectral thinness and effective resistances.
- Strongly Rayleigh distributions, real stable polynomials, and interlacing.
- Effective resistance reducing convex programs.

The tutorial will include open problems and new directions for research.

Part I: Thin trees done three ways (Amin Saberi)

- Introduction
  - Traveling Salesman problem and its variations
  - Linear programming formulation, a brief history and state of the art
- Thin spanning trees
  - Definition, Goddyn's conjecture
  - Implications for the Asymmetric TSP: $O(\log n/\log\log n)$, constant for planar, $\operatorname{poly}\log\log n$ for the integrality gap
- Constructing thin trees by sampling
  - Maximum entropy convex programs
  - $O(\log n/\log\log n)$ approximation for ATSP
  - $3/2-\epsilon$ approximation for TSP
- Spectrally thin trees
  - Definition and relationship with effective resistance
  - A comparison of combinatorial and spectral thinness

Part II: Strongly Rayleigh measures and spectral thinness (Nima Anari)

- Spectral thinness and Kardison-Singer's problem
  - (Recall) Definition of spectrally thin trees
  - The relationship with graph sparsification and Marcus-Spielman-Srivastava's proof of Kadison-Singer's problem
- Real stable polynomials and strongly Rayleigh measures
  - Definitions of real stability and strongly Rayleigh measures with examples
  - Operations preserving real stability and their combinatorial interpretation
  - Applications of the theory of real stable polynomials
- Extension of MSS to strongly Rayleigh measures
  - Mixed characteristic polynomials, interlacing, and barrier arguments
  - Application: sufficient conditions for the existence of spectrally thin trees

Part III: Effective resistance reducing flows (Shayan Oveis Gharan)

- Introduction to electrical flows and effective resistance
- The relationship between edge connectivity and effective resistance
- Effective resistance reducing flows and associated convex programs
- Duality, and the impossibility of reducing the maximum effective resistance of edges
- Reducing the average effective resistance of subsets of edges